```
A=[4 0;0 2;1 1];b=[2 0 11]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
     1     0     1
     0     1     2

    'system NOW consistent'
    'Identify free/basic var'

all variables basic
x_1 BASIC variable
x_2 BASIC variable
p-vector
     1
     2
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 2 & 0 \\ 1 & 1 & 11 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

No. of Columns of A: n = 2 rank(A) = 2 ≠ rank([A b]) = 3

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 17 & 1 & 19 \\ 1 & 5 & 11 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

system NOW consistent
Identify free/basic var

x_1 basic variable
x_2 basic variable

All variables BASIC
Solution is UNIQUE
Empty Null space

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = 1$
$x_2 = 2$

Error vector b-Ax

$$\begin{bmatrix} -2 \\ -4 \\ 8 \end{bmatrix}$$

Least Squares Error ||b-Ax|| 9.1652

```
A=[2,1;4,2;-2,1];b=[3;2;1];linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000         0    0.1000
         0    1.0000    1.2000

    'system NOW consistent'
    'Identify free/basic var'

all variables basic
x_1 BASIC variable
x_2 BASIC variable
p-vector
    0.1000
    1.2000
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 2 \\ -2 & 1 & 1 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

No. of Columns of A: n = 2 rank(A) = 2 ≠ rank([A b]) = 3

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 24 & 8 & 12 \\ 8 & 6 & 8 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 0.1 \\ 0 & 1 & 1.2 \end{bmatrix}$$

system NOW consistent
Identify free/basic var

x_1 basic variable
x_2 basic variable

All variables BASIC
Solution is UNIQUE
Empty Null space

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = 0.1$
$x_2 = 1.2$

Error vector b-Ax

$$\begin{bmatrix} 1.6 \\ -0.8 \\ 0 \end{bmatrix}$$

Least Squares Error ||b-Ax|| 1.7889

```
A=[1,3,4,3];b=1;linearsystem_solverF(A,b)
```

```
     1     3     4     3     1

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
    x2 FREE variable
    x3 FREE variable
    x4 FREE variable
p-vector
     1
     0
     0
     0

v-vector associated with the free variable x2
    -3
     1
     0
     0

v-vector associated with the free variable x3
    -4
     0
     1
     0

v-vector associated with the free variable x4
    -3
     0
     0
     1

Null space v
    -3    -4    -3
     1     0     0
     0     1     0
     0     0     1


ans =

     1     0     0     0
```

Solution of a System of Equations

Augmented Matrix [A b]: 1 3 4 3 1

RREF ([A b]): 1 3 4 3 1

No. of Columns of A: n = 4

rank(A)= rank([A b])= 1

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable

x_2 free variable

x_3 free variable

x_4 free variable

Null Space [v]

-3	-4	-3
1	0	0
0	1	0
0	0	1

rank(A)= rank([A,b]) < n

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2 3 4]

$$\begin{matrix} x_1 = 1 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{matrix} \quad \begin{matrix} -3 \\ 1 \\ 0 \\ 0 \end{matrix} + x_2 \begin{matrix} -4 \\ 0 \\ 1 \\ 0 \end{matrix} + x_3 \begin{matrix} -3 \\ 0 \\ 0 \\ 1 \end{matrix} + x_4$$

Verification: A*p-b $\Rightarrow$

A*p-b must be NULL: 0

Verification: A*v $\Rightarrow$

A*v must be NULL: 0 0 0

```
A=[1 1 0;1 0 -1;0 1 1;-1 01 1];b=[2 5 6 6]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
     1     0     0    -3
     0     1     0     8
     0     0     1    -5

    'system NOW consistent'
    'Identify free/basic var'

all variables basic
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
p-vector
    -3
     8
    -5
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 0 & -1 & 5 \\ 0 & 1 & 1 & 6 \\ -1 & 1 & 1 & 6 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A) = 3 ≠ rank([A b]) = 4

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 3 & 0 & -2 & 1 \\ 0 & 3 & 2 & 14 \\ -2 & 2 & 3 & 7 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

system NOW consistent
Identify free/basic var

x_1 basic variable
x_2 basic variable
x_3 basic variable

All variables BASIC
Solution is UNIQUE
Empty Null space

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = -3$
$x_2 = 8$
$x_3 = -5$

Error vector b-Ax

$$\begin{bmatrix} -3 \\ 3 \\ 3 \\ 0 \end{bmatrix}$$

Least Squares Error ||b-Ax|| 5.1962

```
A=[1,3;-2,-6;3,9];b=[1;0;1];linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000    3.0000    0.2857
         0         0         0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
    x2 FREE variable
p-vector
    0.2857
         0

v-vector associated with the free variable x2
    -3
     1

Null space v
    -3
     1
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

1	3	1
-2	-6	0
3	9	1

RREF ([A b])

1	3	0
0	0	1
0	0	0

No. of Columns of A: n = 2 rank(A) = 1 ≠ rank([A b]) = 2

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

14	42	4
42	126	12

RREF of $[A^T.A \;\; A^T.b]$

1	3	0.29
0	0	0

system consistent

Identify free/basic var x_1 basic variable x_2 free variable

Null space [V] Null Space [v] = $\begin{matrix} -3 \\ 1 \end{matrix}$

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2]

$$\begin{matrix} x_1 = 0.29 \\ x_2 = 0 \end{matrix} + x_2 \begin{matrix} -3 \\ 1 \end{matrix}$$

```
A=[-1,3,2;2,1,3;0,1,1];b=[7 0 -7]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000         0    1.0000   -1.1667
         0    1.0000    1.0000    1.1667
         0         0         0         0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
x_2 BASIC variable
    x3 FREE variable
p-vector
   -1.1667
    1.1667
         0

v-vector associated with the free variable x3
    -1
    -1
     1

Null space v
    -1
    -1
     1
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} -1 & 3 & 2 & 7 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & -7 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A) = 2 ≠ rank([A b]) = 3

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 5 & -1 & 4 & -7 \\ -1 & 11 & 10 & 14 \\ 4 & 10 & 14 & 7 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 1 & -1.17 \\ 0 & 1 & 1 & 1.17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

system consistent

Identify free/basic var

x_1 basic variable

x_2 basic variable

x_3 free variable

Null space [V]

Null Space [v]

$$\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

Solution vector: $[x]=[p]+\Sigma_{[k]} x_{[k]}{}^{*} v_{[k]} \Rightarrow$

with k= [3]

$$\begin{aligned} x_1 &= -1.17 + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \\ x_2 &= 1.17 \\ x_3 &= 0 \end{aligned}$$

```
A=[0 7 1;-7 0 7;-1 7 0;-7 -1 -7;0 -7 -1;7 0 7];
b=[7 0 7 3 2 5]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000         0         0    0.0341
         0    1.0000         0    0.5416
         0         0    1.0000    0.0400

    'system NOW consistent'
    'Identify free/basic var'

all variables basic
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
p-vector
    0.0341
    0.5416
    0.0400
```

```
A=[1,-2,0,1;0,0,1,3];b=[2,1]';linearsystem_solverF(A,b)
```

```
     1    -2     0     1     2
     0     0     1     3     1

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_3 BASIC variable
    x2 FREE variable
    x4 FREE variable
p-vector
     2
     0
     1
     0

v-vector associated with the free variable x2
     2
     1
     0
     0

v-vector associated with the free variable x4
    -1
     0
    -3
     1

Null space v
     2    -1
     1     0
     0    -3
     0     1


ans =

     2     0     1     0
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix}$$

No. of Columns of A: n = 4 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable x_2 free variable

x_3 basic variable x_4 free variable

Null Space [v]

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 0 & -3 \\ 0 & 1 \end{bmatrix}$$

rank(A)= rank([A,b]) < n

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2 4]

$$\begin{matrix} x_1 = 2 \\ x_2 = 0 \\ x_3 = 1 \\ x_4 = 0 \end{matrix} \quad \begin{matrix} 2 \\ 1 \\ 0 \\ 0 \end{matrix} + x_2 \quad + x_4 \quad \begin{matrix} -1 \\ 0 \\ -3 \\ 1 \end{matrix}$$

Verification: A*p-b ⇒

A*p-b must be NULL

$$\begin{matrix} 0 \\ 0 \end{matrix}$$

Verification: A*v ⇒

A*v must be NULL

$$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}$$

```
A=[1,3,4;3,9,7];b=[7,6]';linearsystem_solverF(A,b)
```

```
     1     3     0    -5
     0     0     1     3

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_3 BASIC variable
    x2 FREE variable
p-vector
    -5
     0
     3

v-vector associated with the free variable x2
    -3
     1
     0

Null space v
    -3
     1
     0


ans =

    -5     0     3
```

Solution of a System of Equations

Augmented Matrix [A b]: $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$

RREF ([A b]): $\begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

No. of Columns of A: n = 3 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable

x_3 basic variable

x_2 free variable

Null Space [v]: $\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$

rank(A)= rank([A,b]) < n

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2]

$$\begin{matrix} x_1 = -5 \\ x_2 = 0 \\ x_3 = 3 \end{matrix} \quad \begin{matrix} -3 \\ + x_2 \; 1 \\ 0 \end{matrix}$$

Verification: A*p-b ⇒ $\begin{matrix} 0 \\ 0 \end{matrix}$

A*p-b must be NULL

Verification: A*v ⇒ $\begin{matrix} 0 \\ 0 \end{matrix}$

A*v must be NULL

```
A=[1,-3,7;-2,1,-4;1,2,9];b=[0,0,0]';linearsystem_solverF(A,b)
```

```
     1     0     0     0
     0     1     0     0
     0     0     1     0

system consistent: No PIVOT in the last column of RREF
all variables basic
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
p-vector
     0
     0
     0


ans =

     0     0     0
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & -3 & 7 & 0 \\ -2 & 1 & -4 & 0 \\ 1 & 2 & 9 & 0 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A)= rank([A b])= 3

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

x_1 basic variable
x_2 basic variable
x_3 basic variable

All variables BASIC: UNIQUE Soluti
Empty Null space
rank(A) = rank([A,b]) = n

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

Verification: A*p-b must be NULL: A*p-b ⇒

0
0
0

```
A=[1 3 5;1 1 0;1 1 2;1 3 3];b=[3 5 7 -3]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
     1     0     0    10
     0     1     0    -6
     0     0     1     2

    'system NOW consistent'
    'Identify free/basic var'

all variables basic
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
p-vector
    10
    -6
     2
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 3 & 5 & 3 \\ 1 & 1 & 0 & 5 \\ 1 & 1 & 2 & 7 \\ 1 & 3 & 3 & -3 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A) = 3 ≠ rank([A b]) = 4

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 4 & 8 & 10 & 12 \\ 8 & 20 & 26 & 12 \\ 10 & 26 & 38 & 20 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

system NOW consistent
Identify free/basic var

x_1 basic variable
x_2 basic variable
x_3 basic variable

All variables BASIC
Solution is UNIQUE
Empty Null space

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = 10$
$x_2 = -6$
$x_3 = 2$

Error vector b-Ax

$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

Least Squares Error ||b-Ax|| 2

```
A=[1 -6;1 -2;1 1;1 7];b=[-1 2 1 6]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000         0    2.0000
         0    1.0000    0.5000

    'system NOW consistent'
    'Identify free/basic var'

all variables basic
x_1 BASIC variable
x_2 BASIC variable
p-vector
    2.0000
    0.5000
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 1 & -6 & -1 \\ 1 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 7 & 6 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

No. of Columns of A: n = 2 rank(A) = 2 ≠ rank([A b]) = 3

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 4 & 0 & 8 \\ 0 & 90 & 45 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0.5 \end{bmatrix}$$

system NOW consistent
Identify free/basic var

x_1 basic variable
x_2 basic variable

All variables BASIC
Solution is UNIQUE
Empty Null space

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = 2$
$x_2 = 0.5$

Error vector b-Ax

$$\begin{bmatrix} 0 \\ 1 \\ -1.5 \\ 0.5 \end{bmatrix}$$

Least Squares Error ||b-Ax|| 1.8708

```
A=[1 1 0;1 1 0;1 0 1;1 0 1];b=[1 3 8 2]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
     1     0     1     5
     0     1    -1    -3
     0     0     0     0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
x_2 BASIC variable
    x3 FREE variable
p-vector
     5
    -3
     0

v-vector associated with the free variable x3
    -1
     1
     1

Null space v
    -1
     1
     1
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 3 \\ 1 & 0 & 1 & 8 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A) = 2 ≠ rank([A b]) = 3

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 4 & 2 & 2 & 14 \\ 2 & 2 & 0 & 4 \\ 2 & 0 & 2 & 10 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

system consistent

Identify free/basic var: x_1 basic variable, x_2 basic variable, x_3 free variable

Null space [V]

Null Space [v]

$$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [3]

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

```
A=[1 1 0;1 1 0;1 1 0;1 0 1;1 0 1;1 0 1];b=[7 2 3 6 5 4]';
linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
     1     0     1     5
     0     1    -1    -1
     0     0     0     0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
x_2 BASIC variable
    x3 FREE variable
p-vector
     5
    -1
     0

v-vector associated with the free variable x3
    -1
     1
     1

Null space v
    -1
     1
     1
```

```
A=[0 1 -4;2 -3 2;5 -8 7];b=[8 1 1]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000         0   -5.0000   12.6667
         0    1.0000   -4.0000    7.8333
         0         0         0         0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
x_2 BASIC variable
    x3 FREE variable
p-vector
   12.6667
    7.8333
         0

v-vector associated with the free variable x3
     5
     4
     1

Null space v
     5
     4
     1
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & 7 & 1 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A) = 2 ≠ rank([A b]) = 3

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 29 & -46 & 39 & 7 \\ -46 & 74 & -66 & -3 \\ 39 & -66 & 69 & -23 \end{bmatrix}$$

RREF of $[A^T.A \;\; A^T.b]$

$$\begin{bmatrix} 1 & 0 & -5 & 12.67 \\ 0 & 1 & -4 & 7.83 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

system consistent

Identify free/basic var

x_1 basic variable

x_2 basic variable

x_3 free variable

Null space [V]

Null Space [v]

$$\begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

Solution vector: $[x]=[p]+\Sigma_{[k]} x_{[k]}{}^{*} v_{[k]} \Rightarrow$

with k= [3]

$$\begin{aligned} x_1 &= 12.67 + x_3 \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} \\ x_2 &= 7.83 \\ x_3 &= 0 \end{aligned}$$

```
A=[ 1 7 3;0 1 0;0 1 -1;0 0 1];
b=[-4 1 3 -2]';
linearsystem_solverF(A,b)
```

```
     1     0     0    -5
     0     1     0     1
     0     0     1    -2
     0     0     0     0

system consistent: No PIVOT in the last column of RREF
all variables basic
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
p-vector
    -5
     1
    -2


ans =

    -5     1    -2
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A)= rank([A b])= 3

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

x_1 basic variable
x_2 basic variable
x_3 basic variable

All variables BASIC: UNIQUE Soluti
Empty Null space
rank(A) = rank([A,b]) = n

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = -5$
$x_2 = 1$
$x_3 = -2$

Verification: A*p-b must be NULL: A*p-b ⇒

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
A=[ 1 2; 2 4]; b=[2 2]';linearsystem_solverF(A,b)
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000    2.0000    1.2000
         0         0         0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
    x2 FREE variable
p-vector
    1.2000
         0

v-vector associated with the free variable x2
    -2
     1

Null space v
    -2
```

```
     1
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]: $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 2 \end{bmatrix}$ RREF ([A b]): $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

No. of Columns of A: n = 2 rank(A) = 1 ≠ rank([A b]) = 2

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$: $\begin{bmatrix} 5 & 10 & 6 \\ 10 & 20 & 12 \end{bmatrix}$ RREF of $[A^T.A \;\; A^T.b]$: $\begin{bmatrix} 1 & 2 & 1.2 \\ 0 & 0 & 0 \end{bmatrix}$

system consistent

Identify free/basic var: x_1 basic variable, x_2 free variable

Null space [V]: Null Space [v] $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

Solution vector: $[x]=[p]+\Sigma_{[k]} x_{[k]}{}^{*} v_{[k]} \Rightarrow$

with k= [2]

$x_1 = 1.2$, $x_2 = 0$ $+ x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

```
A=[1 2; 3 4]; b=[2 2]';linearsystem_solverF(A,b)
```

```
     1     0    -2
     0     1     2

system consistent: No PIVOT in the last column of RREF
all variables basic
x_1 BASIC variable
x_2 BASIC variable
p-vector
    -2
     2


ans =

    -2     2
```

Solution of a System of Equations

Augmented Matrix [A b]: 1 2 2 / 3 4 2

RREF ([A b]): 1 0 -2 / 0 1 2

No. of Columns of A: n = 2 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var: x_1 basic variable; x_2 basic variable

All variables BASIC: UNIQUE Soluti
Empty Null space
rank(A) = rank([A,b]) = n

Solution vector written as [x]=[p] ⇒ UNIQUE Solution

$x_1 = -2$

$x_2 = 2$

Verification: A*p-b must be NULL: A*p-b ⇒ 0 0

```
A=[1 0 1 0;0 1 -1 -1;1 1 0 0;0 0 0 1];b=[20 0 80 60]';
linearsystem_solverF(A,b);
```

```
     1     0     1     0    20
     0     1    -1     0    60
     0     0     0     1    60
     0     0     0     0     0

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_2 BASIC variable
x_4 BASIC variable
    x3 FREE variable
p-vector
    20
    60
     0
    60
```

```
v-vector associated with the free variable x3
    -1
     1
     1
     0

Null space v
    -1
     1
     1
     0


ans =

    20    60     0    60
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of Columns of A: n = 4

rank(A)= rank([A b])= 3

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

x_1 basic variable

x_2 basic variable

x_3 free variable

x_4 basic variable

Null space [V]

Null Space [v]

$$\begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

rank(A)= rank([A,b]) < n

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [3]

$$\begin{aligned} x_1 &= 20 \\ x_2 &= 60 \\ x_3 &= 0 \\ x_4 &= 60 \end{aligned} \qquad + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Verification: A*p-b ⇒

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

A*p-b must be NULL

Verification: A*v ⇒

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

A*v must be NULL

```
A=[3 0 0 -1;1 0 -2 0;4 6 -8 -3;0 1 -3 0;0 2 0 -1];b=[0 0 0 0 0]';
linearsystem_solverF(A,b);
```

```
    1.0000         0         0   -0.3333         0
         0    1.0000         0   -0.5000         0
         0         0    1.0000   -0.1667         0
         0         0         0         0         0
         0         0         0         0         0

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
    x4 FREE variable
p-vector
     0
     0
     0
     0

v-vector associated with the free variable x4
    0.3300
    0.5000
    0.1700
    1.0000

Null space v
    0.3300
    0.5000
    0.1700
    1.0000
```

```
ans =

     0     0     0     0
```

```
A=[2 -4 0 2;0 0 2 6];b=[4;2];linearsystem_solverF(A,b);
```

```
     1    -2     0     1     2
     0     0     1     3     1

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_3 BASIC variable
    x2 FREE variable
    x4 FREE variable
p-vector
     2
     0
     1
     0

v-vector associated with the free variable x2
     2
     1
     0
     0

v-vector associated with the free variable x4
    -1
     0
    -3
     1

Null space v
     2    -1
     1     0
     0    -3
```

```
     0     1

ans =

     2     0     1     0
```

Solution of a System of Equations

Augmented Matrix [A b]: 2 -4 0 2 4 / 0 0 2 6 2

RREF ([A b]): 1 -2 0 1 2 / 0 0 1 3 1

No. of Columns of A: n = 4 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable, x_2 free variable

x_3 basic variable, x_4 free variable

rank(A)= rank([A,b]) < n

Null Space [v]:

2 -1
1 0
0 -3
0 1

Solution vector: $[x]=[p]+\Sigma_{[k]} x_{[k]}^{*} v_{[k]} \Rightarrow$

with k= [2 4]

$$\begin{matrix} x_1 = 2 \\ x_2 = 0 \\ x_3 = 1 \\ x_4 = 0 \end{matrix} = \begin{matrix} 2 \\ 0 \\ 1 \\ 0 \end{matrix} + x_2 \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix} + x_4 \begin{matrix} 0 \\ -3 \\ 1 \end{matrix}$$

Verification: A*p-b ⇒ 0 0

A*p-b must be NULL

Verification: A*v ⇒ 0 0 0 0

A*v must be NULL

```
A=[1 -2 0 1;0 0 1 3;0 0 0 0];b=[2 1 0]';linearsystem_solverF(A,b);
```

```
     1    -2     0     1     2
     0     0     1     3     1
     0     0     0     0     0

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_3 BASIC variable
    x2 FREE variable
    x4 FREE variable
p-vector
     2
     0
     1
     0

v-vector associated with the free variable x2
     2
     1
     0
     0

v-vector associated with the free variable x4
    -1
     0
    -3
     1

Null space v
     2    -1
     1     0
     0    -3
     0     1


ans =

     2     0     1     0
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of Columns of A: n = 4 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable x_2 free variable

x_3 basic variable x_4 free variable

rank(A)= rank([A,b]) < n

Null Space [v]

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 0 & -3 \\ 0 & 1 \end{bmatrix}$$

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2 4]

$$\begin{matrix} x_1 = 2 \\ x_2 = 0 \\ x_3 = 1 \\ x_4 = 0 \end{matrix} \quad \begin{matrix} 2 \\ 1 \\ 0 \\ 0 \end{matrix} + x_2 \begin{matrix} -1 \\ 0 \\ -3 \\ 1 \end{matrix} + x_4$$

Verification: A*p-b ⇒

A*p-b must be NULL

$$\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

Verification: A*v ⇒

A*v must be NULL

$$\begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}$$

```
A =[2 -5 -8;-2 -7 1;4 2 7];b=[0 0 0]';linearsystem_solverF(A,b);
```

```
     1     0     0     0
     0     1     0     0
     0     0     1     0

system consistent: No PIVOT in the last column of RREF
all variables basic
x_1 BASIC variable
x_2 BASIC variable
x_3 BASIC variable
p-vector
     0
     0
     0


ans =

     0     0     0
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 2 & -5 & -8 & 0 \\ -2 & -7 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A)= rank([A b])= 3

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

x_1 basic variable
x_2 basic variable
x_3 basic variable

All variables BASIC: UNIQUE Soluti
Empty Null space
rank(A) = rank([A,b]) = n

Solution vector written as [x]=[p] ⇒
UNIQUE Solution

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

Verification: A*p-b must be NULL: A*p-b ⇒

0
0
0

```
A=[1 3 4 7;3 9 7 6];b=[7 6]';linearsystem_solverF(A,b);
```

```
     1     3     0    -5    -5
     0     0     1     3     3

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_3 BASIC variable
    x2 FREE variable
    x4 FREE variable
p-vector
    -5
     0
     3
     0

v-vector associated with the free variable x2
    -3
     1
     0
     0

v-vector associated with the free variable x4
     5
     0
    -3
     1

Null space v
    -3     5
     1     0
     0    -3
     0     1


ans =

    -5     0     3     0
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 3 & 4 & 7 & 7 \\ 3 & 9 & 7 & 6 & 6 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 3 & 0 & -5 & -5 \\ 0 & 0 & 1 & 3 & 3 \end{bmatrix}$$

No. of Columns of A: n = 4 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable, x_2 free variable

x_3 basic variable, x_4 free variable

rank(A)= rank([A,b]) < n

Null Space [v]

$$\begin{bmatrix} -3 & 5 \\ 1 & 0 \\ 0 & -3 \\ 0 & 1 \end{bmatrix}$$

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2 4]

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 5 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$

Verification: A*p-b ⇒ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

A*p-b must be NULL

Verification: A*v ⇒ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

A*v must be NULL

```
A=[1 3 4;0 0 0; 3 9 7];b=[7 0 6]';linearsystem_solverF(A,b);
```

```
     1     3     0    -5
     0     0     1     3
     0     0     0     0

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_3 BASIC variable
    x2 FREE variable
p-vector
    -5
     0
     3

v-vector associated with the free variable x2
    -3
     1
     0

Null space v
    -3
     1
     0


ans =

    -5     0     3
```

Solution of a System of Equations

Augmented Matrix [A b]

1	3	4	7
0	0	0	0
3	9	7	6

RREF ([A b])

1	3	0	-5
0	0	1	3
0	0	0	0

No. of Columns of A: n = 3 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable

x_3 basic variable

x_2 free variable

rank(A)= rank([A,b]) < n

Null Space [v]

-3
1
0

Solution vector: [x]=[p]+$\Sigma_{[k]}$ $x_{[k]}$* $v_{[k]}$ ⇒

with k= [2]

x_1= -5

x_2= 0

x_3= 3

$+ x_2$

-3
1
0

Verification: A*p-b ⇒

A*p-b must be NULL

0
0
0

Verification: A*v ⇒

A*v must be NULL

0
0
0

```
A=[9 -18 -2;-18 36 4];b=[2 4]';linearsystem_solverF(A,b);
```

```
The Linear System is inconsistent
INCOnSISTENT system: Last column of RRF has a PIVOT
USE LEAST SQUARES SOLUTIONS
    1.0000   -2.0000   -0.2222   -0.1333
         0         0         0         0
         0         0         0         0

    'system consistent'
    'Identify free/basic var'
    'Null space [V]'

free variables exist
x_1 BASIC variable
    x2 FREE variable
    x3 FREE variable
p-vector
   -0.1333
         0
         0

v-vector associated with the free variable x2
     2
     1
     0

v-vector associated with the free variable x3
    0.2200
         0
    1.0000

Null space v
    2.0000    0.2200
    1.0000         0
         0    1.0000
```

Solution of a System of Equations Ax = b

Input Augmented Matrix [A b]

9	-18	-2	2
-18	36	4	4

RREF ([A b])

1	-2	-0.22	0
0	0	0	1

No. of Columns of A: n = 3 rank(A) = 1 ≠ rank([A b]) = 2

INCOnSISTENT (Last column of RRF has a PIVOT): Least Squares Solutions

Augmented Matrix $[A^T.A \;\; A^T.b]$

405	-810	-90	-54
-810	1620	180	108
-90	180	20	12

RREF of $[A^T.A \;\; A^T.b]$

1	-2	-0.22	-0.13
0	0	0	0
0	0	0	0

system consistent
Identify free/basic var
Null space [V]

x_1 basic variable
x_2 free variable
x_3 free variable

Null Space [v]

2	0.22
1	0
0	1

Solution vector: $[x]=[p]+\Sigma_{[k]}\, x_{[k]}{}^{*}\, v_{[k]} \Rightarrow$

with k= [2 3]

$$\begin{matrix} x_1 = \\ x_2 = \\ x_3 = \end{matrix} \begin{matrix} -0.13 \\ 0 \\ 0 \end{matrix} + x_2 \begin{matrix} 2 \\ 1 \\ 0 \end{matrix} + x_3 \begin{matrix} 0.22 \\ 0 \\ 1 \end{matrix}$$

```
A=[1 3 -5;1 4 -8; -3 -7 9];b=[4 7 -6]';linearsystem_solverF(A,b);
```

```
     1     0     4    -5
     0     1    -3     3
     0     0     0     0

system consistent: No PIVOT in the last column of RREF
free variables exist
x_1 BASIC variable
x_2 BASIC variable
    x3 FREE variable
p-vector
    -5
     3
     0

v-vector associated with the free variable x3
    -4
     3
     1

Null space v
    -4
     3
     1

ans =

    -5     3     0
```

Solution of a System of Equations

Augmented Matrix [A b]

$$\begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix}$$

RREF ([A b])

$$\begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of Columns of A: n = 3 rank(A)= rank([A b])= 2

system consistent: No PIVOT in the last column of RREF

Identify free/basic var

Null space [V]

x_1 basic variable

x_2 basic variable

x_3 free variable

Null Space [v]

$$\begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

rank(A)= rank([A,b]) < n

Solution vector: $[x]=[p]+\Sigma_{[k]} x_{[k]}{}^{*} v_{[k]} \Rightarrow$

with k= [3]

$$\begin{aligned} x_1 &= -5 \\ x_2 &= 3 \\ x_3 &= 0 \end{aligned} \quad + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

Verification: A*p-b $\Rightarrow$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A*p-b must be NULL

Verification: A*v $\Rightarrow$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A*v must be NULL

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