```
% a few examples used for MS
```

Complete results of some of the examples presented and one example of a non-defective matrix are presented. These include the outputs of the command window as well.

```
A=[4 1 3;1 1 1;-5 -2 -4]; gen_eigenvector_completef_simple(A)
```

```
Fundamental Matrix X

[ -exp(t),   3*t + 1/2,     t - 1/2]
[       0, (3*t)/2 + 1,         t/2]
[  exp(t),    -(9*t)/2, 1 - (3*t)/2]

Matrix Exponential
[ 2*t + 2*exp(t) - 1, 2*t - exp(t) + 1, 2*t + exp(t) - 1]
[                  t,            t + 1,                t]
[ 2 - 2*exp(t) - 3*t, exp(t) - 3*t - 1, 2 - exp(t) - 3*t]


Matrix Exponential directly from Matlab expm(A*t)
[ 2*t + 2*exp(t) - 1, 2*t - exp(t) + 1, 2*t + exp(t) - 1]
[                  t,            t + 1,                t]
[ 2 - 2*exp(t) - 3*t, exp(t) - 3*t - 1, 2 - exp(t) - 3*t]

Matrix Exponential using inverse Laplace
[ 2*t + 2*exp(t) - 1, 2*t - exp(t) + 1, 2*t + exp(t) - 1]
[                  t,            t + 1,                t]
[ 2 - 2*exp(t) - 3*t, exp(t) - 3*t - 1, 2 - exp(t) - 3*t]
```

Eigenvalues, eigenvectors, generalized eigenvectors, fundamental matrix and matrix exponential of a square matrix A of size [n x n]

⇒ Matrix not defective

Eigenvalues/vectors: $\lambda_1,\ v_1;\ \lambda_2,\ v_2;\ \cdots;\lambda_n, v_n$

Solution set: $x_k = e^{\lambda_k t} v_k,\ k = 1, 2, \cdots, n$

⇒ Matrix defective: An example with two distinct eigenvectors

Eigenvalues & distinct eigenvectors (two): λ_1, v_1 & λ_2, v_2

Eigenvalue (λ_3, algebraic multiplicity of $m = n - 2,\ n \geq 3$)

generalized eigenvectors: $v_3, v_4, \cdots, v_n$

Solution set with distinct eigenvectors: $x_k = v_k e^{\lambda_k t},\ k = 1, 2$

Solution set with eigenvalue λ_3 :

$$x_j = [I_n + t(A - \lambda_3 I_n) + \cdots + [\Gamma(m)]^{-1} t^{m-1} (A - \lambda_3 I_n)^{m-1}] v_j e^{\lambda_3 t}$$

$$j = 3, \cdots, n;\ m = n - 2$$

⇒ Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

⇒ Matrix Exponential $e^{At} = \begin{cases} X(t)[X^{-1}(0)], \text{ eigenvectors/values} \\ L^{-1}[(sI_n - A)^{-1}], \text{ inverse Laplace} \\ expm(At), \text{ directly in Matlab} \end{cases}$

Eigenvalues and Eigenvectors

Input Matrix [A]	Eigenvalues (λ)	Eigenvectors (v)
$\begin{pmatrix} 4 & 1 & 3 \\ 1 & 1 & 1 \\ -5 & -2 & -4 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} -\frac{2}{3} & -1 \\ -\frac{1}{3} & 0 \\ 1 & 1 \end{pmatrix}$

Eigenvalues (3) & Eigenvectors (2) ⇒ Matrix DEFECTIVE

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

Generalized eigenvectors required!

Generalized eigenvectors are solutions of $[\,(A - \lambda I_n)^p\,]\,v = 0$

Unique λ	Algebraic mult. (p)	Geometric mult. (m)	Eigenvectors & Generalized eigenvectors
1	1	1	$\begin{pmatrix} -1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$
0	2	1	

λ = 1
λ=0: p = 2

Col. # 1: Eigenvector
Col. # 2, 3: Generalized Eigenvectors

Fundamental Matrix X(t) of A

Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

$$\begin{pmatrix} -e^t & 3t + \frac{1}{2} & t - \frac{1}{2} \\ 0 & \frac{3t}{2} + 1 & \frac{t}{2} \\ e^t & -\frac{9t}{2} & 1 - \frac{3t}{2} \end{pmatrix}$$

Verification using Matrix Exponential: e^{At}

e^{At}: Eigenvalues and eigenvectors (E_X)

$$\begin{pmatrix} 2t+2e^t-1 & 2t-e^t+1 & 2t+e^t-1 \\ t & t+1 & t \\ 2-2e^t-3t & e^t-3t-1 & 2-e^t-3t \end{pmatrix}$$

e^{At}: Inverse Laplace (E_L)

$$\begin{pmatrix} 2t+2e^t-1 & 2t-e^t+1 & 2t+e^t-1 \\ t & t+1 & t \\ 2-2e^t-3t & e^t-3t-1 & 2-e^t-3t \end{pmatrix}$$

e^{At}: Matlab (E_M)

$$\begin{pmatrix} 2t+2e^t-1 & 2t-e^t+1 & 2t+e^t-1 \\ t & t+1 & t \\ 2-2e^t-3t & e^t-3t-1 & 2-e^t-3t \end{pmatrix}$$

ProoF: E_X - $E_L \Rightarrow \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$ ProoF: E_X - $E_M \Rightarrow \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}$

Eigenvalues and Eigenvectors (SUMMARY)

Input Matrix A

4	1	3
1	1	1
-5	-2	-4

Eigenvalues

0
0
1

Eigenvectors

-0.67	-1
-0.33	0
1	1

Eigenvalues (3) & Eigenvectors (2) ⇒ Matrix DEFECTIVE:
Generalized eigenvectors required!

Eigenvectors & Generalized eigenvectors

-1	0.5	-0.5
0	1	0
1	0	1

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

⇒ Generalized Eigenvectors are solutions of $[\,(A - \lambda I_n)^p\,]\,v = 0$

λ = 1

λ=0: p = 2

Col. # 1: Eigenvector

Col. # 2, 3: Generalized Eigenvectors

```
A=[2,-1,0,1;0,2,1,-1;0,0,3,2;0,0,0,3];gen_eigenvector_completef_simple(A)
```

```
Fundamental Matrix X

[ exp(2*t), -t*exp(2*t), -exp(3*t), -2*exp(3*t)*(t - 3)]
[        0,    exp(2*t),  exp(3*t),  exp(3*t)*(2*t - 3)]
[        0,           0,  exp(3*t),        2*t*exp(3*t)]
[        0,           0,         0,            exp(3*t)]

Matrix Exponential
[ exp(2*t), -t*exp(2*t), exp(2*t)*(t - exp(t) + 1), -exp(2*t)*(3*t - 6*exp(t) + 2*t*exp(t) + 6)]
[        0,    exp(2*t),     exp(2*t)*(exp(t) - 1),        exp(2*t)*(2*t*exp(t) - 3*exp(t) + 3)]
[        0,           0,                  exp(3*t),                                2*t*exp(3*t)]
[        0,           0,                         0,                                    exp(3*t)]


Matrix Exponential directly from Matlab expm(A*t)
[ exp(2*t), -t*exp(2*t), exp(2*t)*(t - exp(t) + 1), -exp(2*t)*(3*t - 6*exp(t) + 2*t*exp(t) + 6)]
[        0,    exp(2*t),     exp(2*t)*(exp(t) - 1),        exp(2*t)*(2*t*exp(t) - 3*exp(t) + 3)]
[        0,           0,                  exp(3*t),                                2*t*exp(3*t)]
[        0,           0,                         0,                                    exp(3*t)]

Matrix Exponential using inverse Laplace
[ exp(2*t), -t*exp(2*t), exp(2*t)*(t - exp(t) + 1), -exp(2*t)*(3*t - 6*exp(t) + 2*t*exp(t) + 6)]
[        0,    exp(2*t),     exp(2*t)*(exp(t) - 1),        exp(2*t)*(2*t*exp(t) - 3*exp(t) + 3)]
[        0,           0,                  exp(3*t),                                2*t*exp(3*t)]
[        0,           0,                         0,                                    exp(3*t)]
```

Eigenvalues, eigenvectors, generalized eigenvectors, fundamental matrix and matrix exponential of a square matrix A of size [n x n]

⇒ Matrix not defective

Eigenvalues/vectors: $\lambda_1,\ v_1;\ \lambda_2,\ v_2;\ \cdots;\lambda_n, v_n$

Solution set: $x_k = e^{\lambda_k t} v_k,\ k = 1, 2, \cdots, n$

⇒ Matrix defective: An example with two distinct eigenvectors

Eigenvalues & distinct eigenvectors (two): λ_1, v_1 & λ_2, v_2

Eigenvalue (λ_3, algebraic multiplicity of $m = n - 2,\ n \geq 3$)

generalized eigenvectors: $v_3, v_4, \cdots, v_n$

Solution set with distinct eigenvectors: $x_k = v_k e^{\lambda_k t},\ k = 1, 2$

Solution set with eigenvalue λ_3 :

$$x_j = [I_n + t(A - \lambda_3 I_n) + \cdots + [\Gamma(m)]^{-1} t^{m-1} (A - \lambda_3 I_n)^{m-1}] v_j e^{\lambda_3 t}$$
$$j = 3, \cdots, n;\ m = n - 2$$

⇒ Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

⇒ Matrix Exponential $e^{At} = \begin{cases} X(t)[X^{-1}(0)], \text{ eigenvectors/values} \\ L^{-1}[(sI_n - A)^{-1}], \text{ inverse Laplace} \\ expm(At), \text{ directly in Matlab} \end{cases}$

Eigenvalues and Eigenvectors

Input Matrix [A]

$$\begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

Eigenvalues (λ)

$$\begin{pmatrix} 2 \\ 2 \\ 3 \\ 3 \end{pmatrix}$$

Eigenvectors (v)

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Eigenvalues (4) & Eigenvectors (2) ⇒ Matrix DEFECTIVE

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

Generalized eigenvectors required!

Generalized eigenvectors are solutions of $[(A-\lambda I_n)^p]\,v = 0$

Unique λ	Algebraic mult. (p)	Geometric mult. (m)
2	2	1
3	2	1

Generalized eigenvectors (all)

$$\begin{pmatrix} 1 & 0 & -1 & 6 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

λ=2: p = 2, m = 1

λ=3: p = 2, m = 1

Fundamental Matrix X(t) of A

Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

$$\begin{pmatrix} e^{2t} & -t\,e^{2t} & -e^{3t} & -2\,e^{3t}\,(t-3) \\ 0 & e^{2t} & e^{3t} & e^{3t}\,(2\,t-3) \\ 0 & 0 & e^{3t} & 2\,t\,e^{3t} \\ 0 & 0 & 0 & e^{3t} \end{pmatrix}$$

Verification using Matrix Exponential: e^{At}

e^{At}: Eigenvalues and eigenvectors (E_X)

$$\begin{pmatrix} e^{2t} & -t\,e^{2t} & e^{2t}\,(t - e^t + 1) & -e^{2t}\,(3\,t - 6\,e^t + 2\,t\,e^t + 6) \\ 0 & e^{2t} & e^{2t}\,(e^t - 1) & e^{2t}\,(2\,t\,e^t - 3\,e^t + 3) \\ 0 & 0 & e^{3t} & 2\,t\,e^{3t} \\ 0 & 0 & 0 & e^{3t} \end{pmatrix}$$

e^{At}: Inverse Laplace (E_L)

$$\begin{pmatrix} e^{2t} & -t\,e^{2t} & e^{2t}\,(t - e^t + 1) & -e^{2t}\,(3\,t - 6\,e^t + 2\,t\,e^t + 6) \\ 0 & e^{2t} & e^{2t}\,(e^t - 1) & e^{2t}\,(2\,t\,e^t - 3\,e^t + 3) \\ 0 & 0 & e^{3t} & 2\,t\,e^{3t} \\ 0 & 0 & 0 & e^{3t} \end{pmatrix}$$

e^{At}: Matlab (E_M)

$$\begin{pmatrix} e^{2t} & -t\,e^{2t} & e^{2t}\,(t - e^t + 1) & -e^{2t}\,(3\,t - 6\,e^t + 2\,t\,e^t + 6) \\ 0 & e^{2t} & e^{2t}\,(e^t - 1) & e^{2t}\,(2\,t\,e^t - 3\,e^t + 3) \\ 0 & 0 & e^{3t} & 2\,t\,e^{3t} \\ 0 & 0 & 0 & e^{3t} \end{pmatrix}$$

ProoF: $E_X - E_L \Rightarrow \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ ProoF: $E_X - E_M \Rightarrow \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

Eigenvalues and Eigenvectors (SUMMARY)

Input Matrix A

$$\begin{matrix} 2 & -1 & 0 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{matrix}$$

Eigenvalues

$$\begin{matrix} 2 \\ 2 \\ 3 \\ 3 \end{matrix}$$

Eigenvectors

$$\begin{matrix} 1 & -1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

Eigenvalues (4) & Eigenvectors (2) $\Rightarrow$ Matrix DEFECTIVE: Generalized eigenvectors required!

Generalized eigenvectors (all)

$$\begin{matrix} 1 & 0 & -1 & 6 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

$\Rightarrow$ Generalized Eigenvectors are solutions of $[\,(A-\lambda I_n)^p\,]\,v=0$

λ=2: p = 2, m = 1

λ=3: p = 2, m = 1

```
A=[1,2,0,1;0,1,0,1;0,0,2,2;0,0,0,1];gen_eigenvector_completef_simple(A)
```

```
Fundamental Matrix X

[        0, exp(t), 2*t*exp(t), t*exp(t)*(t + 1)]
[        0,      0,     exp(t),         t*exp(t)]
[ exp(2*t),      0,          0,        -2*exp(t)]
[        0,      0,          0,           exp(t)]

Matrix Exponential
[ exp(t), 2*t*exp(t),        0,      t*exp(t)*(t + 1)]
[      0,     exp(t),        0,              t*exp(t)]
[      0,          0, exp(2*t), 2*exp(t)*(exp(t) - 1)]
[      0,          0,        0,                exp(t)]


Matrix Exponential directly from Matlab expm(A*t)
[ exp(t), 2*t*exp(t),        0,      t*exp(t)*(t + 1)]
[      0,     exp(t),        0,              t*exp(t)]
[      0,          0, exp(2*t), 2*exp(t)*(exp(t) - 1)]
[      0,          0,        0,                exp(t)]

Matrix Exponential using inverse Laplace
[ exp(t), 2*t*exp(t),        0,      t*exp(t)*(t + 1)]
[      0,     exp(t),        0,              t*exp(t)]
[      0,          0, exp(2*t), 2*exp(t)*(exp(t) - 1)]
[      0,          0,        0,                exp(t)]
```

Eigenvalues, eigenvectors, generalized eigenvectors, fundamental matrix and matrix exponential of a square matrix A of size [n x n]

⇒ Matrix not defective

Eigenvalues/vectors: $\lambda_1,\ v_1;\ \lambda_2,\ v_2;\ \cdots;\lambda_n, v_n$

Solution set: $x_k = e^{\lambda_k t} v_k,\ k = 1, 2, \cdots, n$

⇒ Matrix defective: An example with two distinct eigenvectors

Eigenvalues & distinct eigenvectors (two): λ_1, v_1 & λ_2, v_2

Eigenvalue (λ_3, algebraic multiplicity of $m = n - 2,\ n \geq 3$)

generalized eigenvectors: $v_3, v_4, \cdots, v_n$

Solution set with distinct eigenvectors: $x_k = v_k e^{\lambda_k t},\ k = 1, 2$

Solution set with eigenvalue λ_3 :

$$x_j = [I_n + t(A - \lambda_3 I_n) + \cdots + [\Gamma(m)]^{-1} t^{m-1} (A - \lambda_3 I_n)^{m-1}] v_j e^{\lambda_3 t}$$
$$j = 3, \cdots, n;\ m = n - 2$$

⇒ Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

⇒ Matrix Exponential $e^{At} = \begin{cases} X(t)[X^{-1}(0)], \text{ eigenvectors/values} \\ L^{-1}[(sI_n - A)^{-1}], \text{ inverse Laplace} \\ expm(At), \text{ directly in Matlab} \end{cases}$

Eigenvalues and Eigenvectors

Input Matrix [A]

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Eigenvalues (λ)

$$\begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Eigenvectors (v)

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Eigenvalues (4) & Eigenvectors (2) ⇒ Matrix DEFECTIVE

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

Generalized eigenvectors required!

Generalized eigenvectors are solutions of $[\,(A - \lambda I_n)^p\,]\,v = 0$

Unique λ	Algebraic mult. (p)	Geometric mult. (m)
2	1	1
1	3	1

Eigenvectors & Generalized eigenvectors

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

λ = 2
λ=1: p = 3

Col. # 1: Eigenvector
Col. # 2, 3, 4: Generalized Eigenvectors

Fundamental Matrix X(t) of A

Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

$$\begin{pmatrix} 0 & e^{t} & 2\,t\,e^{t} & t\,e^{t}\,(t+1) \\ 0 & 0 & e^{t} & t\,e^{t} \\ e^{2\,t} & 0 & 0 & -2\,e^{t} \\ 0 & 0 & 0 & e^{t} \end{pmatrix}$$

Verification using Matrix Exponential: e^{At}

e^{At}: Eigenvalues and eigenvectors (E_X)

$$\begin{pmatrix} e^t & 2\,t\,e^t & 0 & t\,e^t\,(t+1) \\ 0 & e^t & 0 & t\,e^t \\ 0 & 0 & e^{2\,t} & 2\,e^t\,(e^t-1) \\ 0 & 0 & 0 & e^t \end{pmatrix}$$

e^{At}: Inverse Laplace (E_L)

$$\begin{pmatrix} e^t & 2\,t\,e^t & 0 & t\,e^t\,(t+1) \\ 0 & e^t & 0 & t\,e^t \\ 0 & 0 & e^{2\,t} & 2\,e^t\,(e^t-1) \\ 0 & 0 & 0 & e^t \end{pmatrix}$$

e^{At}: Matlab (E_M)

$$\begin{pmatrix} e^t & 2\,t\,e^t & 0 & t\,e^t\,(t+1) \\ 0 & e^t & 0 & t\,e^t \\ 0 & 0 & e^{2\,t} & 2\,e^t\,(e^t-1) \\ 0 & 0 & 0 & e^t \end{pmatrix}$$

ProoF: E_X - $E_L \Rightarrow \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ ProoF: E_X - $E_M \Rightarrow \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

Eigenvalues and Eigenvectors (SUMMARY)

Input Matrix A

1	2	0	1
0	1	0	1
0	0	2	2
0	0	0	1

Eigenvalues

2
1
1
1

Eigenvectors

0	1
0	0
1	0
0	0

Eigenvalues (4) & Eigenvectors (2) ⇒ Matrix DEFECTIVE:
Generalized eigenvectors required!

Eigenvectors & Generalized eigenvectors

0	1	0	0
0	0	1	0
1	0	0	-2
0	0	0	1

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

⇒ Generalized Eigenvectors are solutions of $[\,(A - \lambda I_n)^p\,]\, v = 0$

λ = 2

λ=1: p = 3

Col. # 1: Eigenvector

Col. # 2, 3, 4: Generalized Eigenvectors

```
A=[0 1 0 0;0 0 1 0;0 0 0 1;-1 0 -2 0];gen_eigenvector_completef_simple(A)
```

```
Fundamental Matrix X

[ - 3*cos(t) - t*sin(t), 3*sin(t) - t*cos(t), 2*sin(t) - t*cos(t), 2*cos(t) + t*sin(t)]
[   2*sin(t) - t*cos(t), 2*cos(t) + t*sin(t),   cos(t) + t*sin(t),   t*cos(t) - sin(t)]
[     cos(t) + t*sin(t),   t*cos(t) - sin(t),            t*cos(t),           -t*sin(t)]
[              t*cos(t),           -t*sin(t),   cos(t) - t*sin(t), - sin(t) - t*cos(t)]

Matrix Exponential
[     cos(t) + (t*sin(t))/2, (3*sin(t))/2 - (t*cos(t))/2,                  (t*sin(t))/2, sin(t)/2
- (t*cos(t))/2]
[   (t*cos(t))/2 - sin(t)/2,       cos(t) + (t*sin(t))/2,       sin(t)/2 + (t*cos(t))/2,
(t*sin(t))/2]
[             -(t*sin(t))/2,     (t*cos(t))/2 - sin(t)/2,         cos(t) - (t*sin(t))/2, sin(t)/2
+ (t*cos(t))/2]
[ - sin(t)/2 - (t*cos(t))/2,               -(t*sin(t))/2, - (3*sin(t))/2 - (t*cos(t))/2,   cos(t)
- (t*sin(t))/2]


Matrix Exponential directly from Matlab expm(A*t)
[     cos(t) + (t*sin(t))/2, (3*sin(t))/2 - (t*cos(t))/2,                  (t*sin(t))/2, sin(t)/2
- (t*cos(t))/2]
[   (t*cos(t))/2 - sin(t)/2,       cos(t) + (t*sin(t))/2,       sin(t)/2 + (t*cos(t))/2,
(t*sin(t))/2]
[             -(t*sin(t))/2,     (t*cos(t))/2 - sin(t)/2,         cos(t) - (t*sin(t))/2, sin(t)/2
+ (t*cos(t))/2]
[ - sin(t)/2 - (t*cos(t))/2,               -(t*sin(t))/2, - (3*sin(t))/2 - (t*cos(t))/2,   cos(t)
- (t*sin(t))/2]

Matrix Exponential using inverse Laplace
[     cos(t) + (t*sin(t))/2, (3*sin(t))/2 - (t*cos(t))/2,                  (t*sin(t))/2, sin(t)/2
- (t*cos(t))/2]
[   (t*cos(t))/2 - sin(t)/2,       cos(t) + (t*sin(t))/2,       sin(t)/2 + (t*cos(t))/2,
(t*sin(t))/2]
[             -(t*sin(t))/2,     (t*cos(t))/2 - sin(t)/2,         cos(t) - (t*sin(t))/2, sin(t)/2
+ (t*cos(t))/2]
[ - sin(t)/2 - (t*cos(t))/2,               -(t*sin(t))/2, - (3*sin(t))/2 - (t*cos(t))/2,   cos(t)
- (t*sin(t))/2]
```

Eigenvalues, eigenvectors, generalized eigenvectors, fundamental matrix and matrix exponential of a square matrix A of size [n x n]

⇒ Matrix not defective

Eigenvalues/vectors: $\lambda_1,\ v_1;\ \lambda_2,\ v_2;\ \cdots;\lambda_n, v_n$

Solution set: $x_k = e^{\lambda_k t} v_k,\ k = 1, 2, \cdots, n$

⇒ Matrix defective: An example with two distinct eigenvectors

Eigenvalues & distinct eigenvectors (two): λ_1, v_1 & λ_2, v_2

Eigenvalue (λ_3, algebraic multiplicity of $m = n - 2,\ n \geq 3$)

generalized eigenvectors: $v_3, v_4, \cdots, v_n$

Solution set with distinct eigenvectors: $x_k = v_k e^{\lambda_k t},\ k = 1, 2$

Solution set with eigenvalue λ_3 :

$$x_j = [I_n + t(A - \lambda_3 I_n) + \cdots + [\Gamma(m)]^{-1} t^{m-1} (A - \lambda_3 I_n)^{m-1}] v_j e^{\lambda_3 t}$$
$$j = 3, \cdots, n;\ m = n - 2$$

⇒ Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

⇒ Matrix Exponential $e^{At} = \begin{cases} X(t)[X^{-1}(0)], \text{ eigenvectors/values} \\ L^{-1}[(sI_n - A)^{-1}], \text{ inverse Laplace} \\ expm(At), \text{ directly in Matlab} \end{cases}$

Eigenvalues and Eigenvectors

Input Matrix [A] Eigenvalues (λ) Eigenvectors (v)

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -2 & 0 \end{pmatrix} \quad \begin{pmatrix} -i \\ -i \\ i \\ i \end{pmatrix} \quad \begin{pmatrix} -i & i \\ -1 & -1 \\ i & -i \\ 1 & 1 \end{pmatrix}$$

Eigenvalues (4) & Eigenvectors (2) ⇒ Matrix DEFECTIVE

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

Generalized eigenvectors required!

Generalized eigenvectors are solutions of $[\,(A-\lambda I_n)^p\,]\,v = 0$

Unique λ	Algebraic mult. (p)	Geometric mult. (m)
0-1i	2	1
0+1i	2	1

Generalized eigenvectors (all)

$$\begin{pmatrix} -3 & 2i & -3 & -2i \\ 2i & 1 & -2i & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Complex Equal Pairs: p = 2

Fundamental Matrix X(t) of A

Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

$$\begin{pmatrix} -3\cos(t) - t\sin(t) & 3\sin(t) - t\cos(t) & 2\sin(t) - t\cos(t) & 2\cos(t) + t\sin(t) \\ 2\sin(t) - t\cos(t) & 2\cos(t) + t\sin(t) & \cos(t) + t\sin(t) & t\cos(t) - \sin(t) \\ \cos(t) + t\sin(t) & t\cos(t) - \sin(t) & t\cos(t) & -t\sin(t) \\ t\cos(t) & -t\sin(t) & \cos(t) - t\sin(t) & -\sin(t) - t\cos(t) \end{pmatrix}$$

Verification using Matrix Exponential: e^{At}

e^{At}: Eigenvalues and eigenvectors (E_X)

$$\begin{pmatrix} \cos(t)+\frac{t\sin(t)}{2} & \frac{3\sin(t)}{2}-\frac{t\cos(t)}{2} & \frac{t\sin(t)}{2} & \frac{\sin(t)}{2}-\frac{t\cos(t)}{2} \\ \frac{t\cos(t)}{2}-\frac{\sin(t)}{2} & \cos(t)+\frac{t\sin(t)}{2} & \frac{\sin(t)}{2}+\frac{t\cos(t)}{2} & \frac{t\sin(t)}{2} \\ -\frac{t\sin(t)}{2} & \frac{t\cos(t)}{2}-\frac{\sin(t)}{2} & \cos(t)-\frac{t\sin(t)}{2} & \frac{\sin(t)}{2}+\frac{t\cos(t)}{2} \\ -\frac{\sin(t)}{2}-\frac{t\cos(t)}{2} & -\frac{t\sin(t)}{2} & -\frac{3\sin(t)}{2}-\frac{t\cos(t)}{2} & \cos(t)-\frac{t\sin(t)}{2} \end{pmatrix}$$

e^{At}: Inverse Laplace (E_L)

$$\begin{pmatrix} \cos(t)+\frac{t\sin(t)}{2} & \frac{3\sin(t)}{2}-\frac{t\cos(t)}{2} & \frac{t\sin(t)}{2} & \frac{\sin(t)}{2}-\frac{t\cos(t)}{2} \\ \frac{t\cos(t)}{2}-\frac{\sin(t)}{2} & \cos(t)+\frac{t\sin(t)}{2} & \frac{\sin(t)}{2}+\frac{t\cos(t)}{2} & \frac{t\sin(t)}{2} \\ -\frac{t\sin(t)}{2} & \frac{t\cos(t)}{2}-\frac{\sin(t)}{2} & \cos(t)-\frac{t\sin(t)}{2} & \frac{\sin(t)}{2}+\frac{t\cos(t)}{2} \\ -\frac{\sin(t)}{2}-\frac{t\cos(t)}{2} & -\frac{t\sin(t)}{2} & -\frac{3\sin(t)}{2}-\frac{t\cos(t)}{2} & \cos(t)-\frac{t\sin(t)}{2} \end{pmatrix}$$

e^{At}: Matlab (E_M)

$$\begin{pmatrix} \cos(t)+\frac{t\sin(t)}{2} & \frac{3\sin(t)}{2}-\frac{t\cos(t)}{2} & \frac{t\sin(t)}{2} & \frac{\sin(t)}{2}-\frac{t\cos(t)}{2} \\ \frac{t\cos(t)}{2}-\frac{\sin(t)}{2} & \cos(t)+\frac{t\sin(t)}{2} & \frac{\sin(t)}{2}+\frac{t\cos(t)}{2} & \frac{t\sin(t)}{2} \\ -\frac{t\sin(t)}{2} & \frac{t\cos(t)}{2}-\frac{\sin(t)}{2} & \cos(t)-\frac{t\sin(t)}{2} & \frac{\sin(t)}{2}+\frac{t\cos(t)}{2} \\ -\frac{\sin(t)}{2}-\frac{t\cos(t)}{2} & -\frac{t\sin(t)}{2} & -\frac{3\sin(t)}{2}-\frac{t\cos(t)}{2} & \cos(t)-\frac{t\sin(t)}{2} \end{pmatrix}$$

ProoF: $E_X - E_L \Rightarrow \begin{matrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{matrix}$ ProoF: $E_X - E_M \Rightarrow \begin{matrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{matrix}$

Eigenvalues and Eigenvectors (SUMMARY)

Input Matrix A

0	1	0	0
0	0	1	0
0	0	0	1
-1	0	-2	0

Eigenvalues		Eigenvectors		
	0-1i		0-1i	0+1i
	0-1i		-1+0i	-1+0i
	0+1i		0+1i	0-1i
	0+1i		1+0i	1+0i

Eigenvalues (4) & Eigenvectors (2) $\Rightarrow$ Matrix DEFECTIVE:
Generalized eigenvectors required!

Generalized eigenvectors (all)

-3+0i	0+2i	-3+0i	0-2i
0+2i	1+0i	0-2i	1+0i
1+0i	0+0i	1+0i	0+0i
0+0i	1+0i	0+0i	1+0i

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

$\Rightarrow$ Generalized Eigenvectors are solutions of $[\,(A - \lambda I_n)^p\,]\,v = 0$

Complex Equal Pairs: p = 2

```
A=[-1 -4 0 0; 1 3 0 0; 1 2 1 0;0 1 0 1];gen_eigenvector_completef_simple(A)
```

```
Fundamental Matrix X

[ -exp(t)*(2*t - 1),      -4*t*exp(t),      0,      0]
[          t*exp(t), exp(t)*(2*t + 1),      0,      0]
[          t*exp(t),       2*t*exp(t), exp(t),      0]
[    (t^2*exp(t))/2, t*exp(t)*(t + 1),      0, exp(t)]

Matrix Exponential
[ -exp(t)*(2*t - 1),      -4*t*exp(t),      0,      0]
[          t*exp(t), exp(t)*(2*t + 1),      0,      0]
[          t*exp(t),       2*t*exp(t), exp(t),      0]
[    (t^2*exp(t))/2, t*exp(t)*(t + 1),      0, exp(t)]


Only a single value, Algebraic multiplicity = number of columns of A

X(0) = must be an identity matrix

[ 1, 0, 0, 0]
[ 0, 1, 0, 0]
[ 0, 0, 1, 0]
[ 0, 0, 0, 1]

Fundamental matrix = matrix exponential

[ 0, 0, 0, 0]
[ 0, 0, 0, 0]
[ 0, 0, 0, 0]
[ 0, 0, 0, 0]


Matrix Exponential directly from Matlab expm(A*t)
[ -exp(t)*(2*t - 1),      -4*t*exp(t),      0,      0]
[          t*exp(t), exp(t)*(2*t + 1),      0,      0]
[          t*exp(t),       2*t*exp(t), exp(t),      0]
[    (t^2*exp(t))/2, t*exp(t)*(t + 1),      0, exp(t)]

Matrix Exponential using inverse Laplace
[ -exp(t)*(2*t - 1),      -4*t*exp(t),      0,      0]
[          t*exp(t), exp(t)*(2*t + 1),      0,      0]
[          t*exp(t),       2*t*exp(t), exp(t),      0]
[    (t^2*exp(t))/2, t*exp(t)*(t + 1),      0, exp(t)]
```

Eigenvalues, eigenvectors, generalized eigenvectors, fundamental matrix and matrix exponential of a square matrix A of size [n x n]

⇒ Matrix not defective

Eigenvalues/vectors: $\lambda_1,\ v_1;\ \lambda_2,\ v_2;\ \cdots;\lambda_n, v_n$

Solution set: $x_k = e^{\lambda_k t} v_k,\ k = 1, 2, \cdots, n$

⇒ Matrix defective: An example with two distinct eigenvectors

Eigenvalues & distinct eigenvectors (two): λ_1, v_1 & λ_2, v_2

Eigenvalue (λ_3, algebraic multiplicity of $m = n - 2,\ n \geq 3$)

generalized eigenvectors: $v_3, v_4, \cdots, v_n$

Solution set with distinct eigenvectors: $x_k = v_k e^{\lambda_k t},\ k = 1, 2$

Solution set with eigenvalue λ_3 :

$$x_j = [I_n + t(A - \lambda_3 I_n) + \cdots + [\Gamma(m)]^{-1} t^{m-1} (A - \lambda_3 I_n)^{m-1}] v_j e^{\lambda_3 t}$$
$$j = 3, \cdots, n;\ m = n - 2$$

⇒ Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

⇒ Matrix Exponential $e^{At} = \begin{cases} X(t)[X^{-1}(0)], \text{ eigenvectors/values} \\ L^{-1}[(sI_n - A)^{-1}], \text{ inverse Laplace} \\ expm(At), \text{ directly in Matlab} \end{cases}$

Eigenvalues and Eigenvectors

Input Matrix [A]	Eigenvalues (λ)	Eigenvectors (v)
$\begin{pmatrix} -1 & -4 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

Eigenvalues (4) & Eigenvectors (2) ⇒ Matrix DEFECTIVE

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

Generalized eigenvectors required!

Generalized eigenvectors are solutions of $[\,(A - \lambda I_n)^p\,]\,v = 0$

Unique λ	**Algebraic mult. (p)**	**Geometric mult. (m)**	**Generalized eigenvectors (all)**
1	4	2	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

λ=1: p = 4

Algebraic multiplicity = No. of columns of A: X(0) = I_4

Fundamental Matrix ≡ Matrix Exponential

Fundamental Matrix X(t) of A

Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

Algebraic multiplicity = number of columns of A: X(0) = I_4

Fundamental Matrix ≡ Matrix Exponential

$$\begin{pmatrix} -e^t\,(2\,t-1) & -4\,t\,e^t & 0 & 0 \\ t\,e^t & e^t\,(2\,t+1) & 0 & 0 \\ t\,e^t & 2\,t\,e^t & e^t & 0 \\ \frac{t^2 e^t}{2} & t\,e^t\,(t+1) & 0 & e^t \end{pmatrix}$$

Verification using Matrix Exponential: e^{At}

e^{At}: Eigenvalues and eigenvectors (E_X), **p = n, X(0) = I_4 ⇒ X(t)≡ e^{At}**

$$\begin{pmatrix} -e^t\,(2t-1) & -4\,t\,e^t & 0 & 0 \\ t\,e^t & e^t\,(2t+1) & 0 & 0 \\ t\,e^t & 2\,t\,e^t & e^t & 0 \\ \frac{t^2\,e^t}{2} & t\,e^t\,(t+1) & 0 & e^t \end{pmatrix}$$

e^{At}: Inverse Laplace (E_L)

$$\begin{pmatrix} -e^t\,(2t-1) & -4\,t\,e^t & 0 & 0 \\ t\,e^t & e^t\,(2t+1) & 0 & 0 \\ t\,e^t & 2\,t\,e^t & e^t & 0 \\ \frac{t^2\,e^t}{2} & t\,e^t\,(t+1) & 0 & e^t \end{pmatrix}$$

e^{At}: Matlab (E_M)

$$\begin{pmatrix} -e^t\,(2t-1) & -4\,t\,e^t & 0 & 0 \\ t\,e^t & e^t\,(2t+1) & 0 & 0 \\ t\,e^t & 2\,t\,e^t & e^t & 0 \\ \frac{t^2\,e^t}{2} & t\,e^t\,(t+1) & 0 & e^t \end{pmatrix}$$

ProoF: X(t)-E_X ⇒ $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ p = n

ProoF: E_X - E_L ⇒ $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

ProoF: E_X - E_M ⇒ $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

Eigenvalues and Eigenvectors (SUMMARY)

Input Matrix A

-1	-4	0	0
1	3	0	0
1	2	1	0
0	1	0	1

Eigenvalues

1
1
1
1

Eigenvectors

0	0
0	0
1	0
0	1

Eigenvalues (4) & Eigenvectors (2) ⇒ Matrix DEFECTIVE:
Generalized eigenvectors required!

Generalized eigenvectors (all)

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Algebraic multiplicity (p) of the eigenvalue λ > geometric multiplicity (m)

⇒ Generalized Eigenvectors are solutions of $[\,(A-\lambda I_n)^p\,]\,v = 0$

λ=1: p = 4

```
A=[1,0,0,0;0,1,0,0;1,4,-3,0;-1,-2,0,-3];gen_eigenvector_completef_simple(A)
```

```
Fundamental Matrix X

[         0,         0, -4*exp(t), -8*exp(t)]
[         0,         0,  2*exp(t),  2*exp(t)]
[ exp(-3*t),         0,    exp(t),         0]
[         0, exp(-3*t),         0,    exp(t)]

Matrix Exponential
[                exp(t),                      0,         0,         0]
[                     0,                 exp(t),         0,         0]
[ exp(t)/4 - exp(-3*t)/4,     exp(t) - exp(-3*t), exp(-3*t),         0]
[ exp(-3*t)/4 - exp(t)/4, exp(-3*t)/2 - exp(t)/2,         0, exp(-3*t)]


Matrix Exponential directly from Matlab expm(A*t)
[                exp(t),                      0,         0,         0]
[                     0,                 exp(t),         0,         0]
[ exp(t)/4 - exp(-3*t)/4,     exp(t) - exp(-3*t), exp(-3*t),         0]
[ exp(-3*t)/4 - exp(t)/4, exp(-3*t)/2 - exp(t)/2,         0, exp(-3*t)]

Matrix Exponential using inverse Laplace
[                exp(t),                      0,         0,         0]
[                     0,                 exp(t),         0,         0]
[ exp(t)/4 - exp(-3*t)/4,     exp(t) - exp(-3*t), exp(-3*t),         0]
[ exp(-3*t)/4 - exp(t)/4, exp(-3*t)/2 - exp(t)/2,         0, exp(-3*t)]
```

Eigenvalues, eigenvectors, generalized eigenvectors, fundamental matrix and matrix exponential of a square matrix A of size [n x n]

⇒ Matrix not defective

Eigenvalues/vectors: $\lambda_1,\ v_1;\ \lambda_2,\ v_2;\ \cdots;\lambda_n, v_n$

Solution set: $x_k = e^{\lambda_k t}v_k,\ k = 1, 2, \cdots, n$

⇒ Matrix defective: An example with two distinct eigenvectors

Eigenvalues & distinct eigenvectors (two): λ_1, v_1 & λ_2, v_2

Eigenvalue (λ_3, algebraic multiplicity of $m = n - 2,\ n \geq 3$)

generalized eigenvectors: $v_3, v_4, \cdots, v_n$

Solution set with distinct eigenvectors: $x_k = v_k e^{\lambda_k t},\ k = 1, 2$

Solution set with eigenvalue λ_3 :

$$x_j = [I_n + t(A - \lambda_3 I_n) + \cdots + [\Gamma(m)]^{-1}t^{m-1}(A - \lambda_3 I_n)^{m-1}]v_j e^{\lambda_3 t}$$
$$j = 3, \cdots, n;\ m = n - 2$$

⇒ Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

⇒ Matrix Exponential $e^{At} = \begin{cases} X(t)[X^{-1}(0)], \text{ eigenvectors/values} \\ L^{-1}[(sI_n - A)^{-1}], \text{ inverse Laplace} \\ expm(At), \text{ directly in Matlab} \end{cases}$

Eigenvalues and Eigenvectors

Input Matrix [A]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{pmatrix}$$

Eigenvalues (λ)

$$\begin{pmatrix} -3 \\ -3 \\ 1 \\ 1 \end{pmatrix}$$

Eigenvectors (v)

$$\begin{pmatrix} 0 & 0 & -4 & -8 \\ 0 & 0 & 2 & 2 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Eigenvalues (4) & Eigenvectors (4): Matrix NOT defective

Fundamental Matrix X(t) of A

Fundamental Matrix of A: $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]$

$$\begin{pmatrix} 0 & 0 & -4\,e^{t} & -8\,e^{t} \\ 0 & 0 & 2\,e^{t} & 2\,e^{t} \\ e^{-3\,t} & 0 & e^{t} & 0 \\ 0 & e^{-3\,t} & 0 & e^{t} \end{pmatrix}$$

Verification using Matrix Exponential: e^{At}

e^{At}: Eigenvalues and eigenvectors (E_X)

$$\begin{pmatrix} e^{t} & 0 & 0 & 0 \\ 0 & e^{t} & 0 & 0 \\ \frac{e^{t}}{4} - \frac{e^{-3t}}{4} & e^{t} - e^{-3t} & e^{-3t} & 0 \\ \frac{e^{-3t}}{4} - \frac{e^{t}}{4} & \frac{e^{-3t}}{2} - \frac{e^{t}}{2} & 0 & e^{-3t} \end{pmatrix}$$

e^{At}: Inverse Laplace (E_L)

$$\begin{pmatrix} e^{t} & 0 & 0 & 0 \\ 0 & e^{t} & 0 & 0 \\ \frac{e^{t}}{4} - \frac{e^{-3t}}{4} & e^{t} - e^{-3t} & e^{-3t} & 0 \\ \frac{e^{-3t}}{4} - \frac{e^{t}}{4} & \frac{e^{-3t}}{2} - \frac{e^{t}}{2} & 0 & e^{-3t} \end{pmatrix}$$

e^{At}: Matlab (E_M)

$$\begin{pmatrix} e^{t} & 0 & 0 & 0 \\ 0 & e^{t} & 0 & 0 \\ \frac{e^{t}}{4} - \frac{e^{-3t}}{4} & e^{t} - e^{-3t} & e^{-3t} & 0 \\ \frac{e^{-3t}}{4} - \frac{e^{t}}{4} & \frac{e^{-3t}}{2} - \frac{e^{t}}{2} & 0 & e^{-3t} \end{pmatrix}$$

ProoF: E_X - $E_L \Rightarrow$ $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ ProoF: E_X - $E_M \Rightarrow$ $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

Eigenvalues and Eigenvectors (SUMMARY)

Input Matrix A

1	0	0	0
0	1	0	0
1	4	-3	0
-1	-2	0	-3

Eigenvalues

-3
-3
1
1

Eigenvectors

0	0	-4	-8
0	0	2	2
1	0	1	0
0	1	0	1

Eigenvalues (4) & Eigenvectors (4): Matrix NOT defective

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